INELASTIC RESPONSE OF ECCENTRIC STRUCTURES

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ABSTRACT

A study on the inelastic responses of a simple two degrees of freedom eccentric structure is carried out to determine the influence of eccentricity and torsional to lateral frequency ratio on the ductility demand and edge displacement of the structure. The resisting elements are taken to be bilinear hysteretic and the El Centro 1940 and Taft 1952 records are used as sources of excitation. It is found that the influence of eccentricity on ductility demand is larger than previously reported by other investigators. An approximate bound is proposed to relate the ductility demand to the excitation level for eccentric structures.

INTRODUCTION

It is well known that both translational and torsional motions are induced in asymmetrical structures when subjected to seismic ground motions. Most studies on the lateral-torsional response problem assumes the system remains in the elastic range [1,2]. Under strong shaking, it is likely that many of the resisting elements will be excited into the inelastic range and the nonlinear hysteretic effect from inelastic action will affect the lateral-torsional responses of the system. The study of inelastic lateral-torsional responses of eccentric structure has received much less attention. Kan and Chopra [3] has carried out a parametric study on the elastic and inelastic responses of a single mass monosymmetric system under horizontal earthquake (El Centro 1940) ground excitation. They found that after yielding the response is primarily in translation and the system behaves more like an inelastic single degree of freedom system. As a result, torsional coupling affects maximum deformation in the inelastic system to a lesser extent compared to corresponding linear elastic system. Irvine and Kountouris [4] studied the bilinear hysteretic response of a simple torsionally unbalanced building consisting of two identical frames supporting a diaphragm subjected to three recorded (El Centro 1940, Taft 1952, Pacoima Dam 1971) and one artificially generated ground motions. They found that the ductility demand on the worst loaded frame is rarely more than 30% greater than the ductility demand in a similar symmetric structure, and the peak ductility and the eccentricity of the structure are only weakly correlated.

In the present investigation, a single mass model with three lateral resisting elements is used to study the torsional-lateral inelastic response of structures subjected to ground motion excitation. The model used here is statically indeterminate, and its torsional stiffness is controlled by the spread of the resisting elements from the mass center. Therefore, the model used is more representative of buildings design in practice than that used by Irvine and Kountouris. The ductility demand and the edge displacement are two response parameters under study. It is shown that the torsional-lateral inelastic response depend on the earthquake record used in a complex manner. Also, the effect of eccentricity has a more pronounced effect on ductility demand and overall behaviour than as described by Irvine and Kountouris.

DESCRIPTION OF MODEL

The structural model used consists of a rigid rectangular deck of mass M and dimensions B by D, supported by three frames A, B and C in the direction of ground excitation. Frame B is located at the center and the other two frames are located at distance x on either side of the center frame as shown in Fig. (1). For simplicity, the contribution of the frames normal to the direction of ground motion to torsional resistance is ignored. The force-displacement relationship for each frame in the y direction is assumed to be bilinear. The slope of the yielding branch is taken as 3% of the initial elastic slope. The elastic slopes of the frames may be different, and their relative values will be adjusted to give rise to different initial eccentricity values to the system. Each frame, however, is assumed to have the same yield displacement U_y. Therefore, a stiffer frame also implies a frame of higher yield strength.

One important parameter on the elastic response of torsionally coupled system is the uncoupled torsional frequency to lateral frequency ratio Ω. The uncoupled lateral period is taken to be 1 second in the current study and the uncoupled torsional period, and hence Ω is varied by changing the frame spacing x.

The system described is basically a two degrees of freedom system. The motions of the system can then be described by a translation $u_y(t)$ of the center of mass and a rotation $u_\theta(t)$ about the center of mass. Although the frames are bilinear, the force-displacement relationship at the center of mass is not straightly bilinear since not all the frames will start yielding at the same translation $u_y(t)$. The force-displacement characteristics at the mass center for a torsionally stiff (Ω = 1.4) and a torsionally weak system (Ω = 0.8) are shown in Fig. 2.

EQUATIONS OF MOTION

The equations of motion for the system subjected to horizontal ground acceleration $\ddot{u}_g(t)$ can be written as

$$\begin{bmatrix} M & 0 \\ 0 & Mr^2 \end{bmatrix} \begin{Bmatrix} \ddot{u}_y \\ \ddot{u}_\theta \end{Bmatrix} + \{F\} = - \begin{Bmatrix} M\,\ddot{u}_g \\ 0 \end{Bmatrix} \qquad (1)$$

where r = radius of gyration about center of mass

{F} = the stiffness-related resisting force vector; in incremental form, it is given by

$$\{\Delta F\} = \begin{bmatrix} \Sigma K_i & \Sigma K_i x_i \\ \Sigma K_i x_i & \Sigma K_i x_i^2 \end{bmatrix} \begin{Bmatrix} \Delta u_y \\ \Delta u_\theta \end{Bmatrix}, \quad (i = A, B \text{ and } C)$$

where K_i is the instantaneous stiffness of frame i as determined from the load-deformation relationship.

The equations of motion can be put into a nondimensional form as follows

$$\begin{bmatrix} 1 & 0 \\ 0 & 4\bar{r}^2 \end{bmatrix} \begin{Bmatrix} \ddot{z} \\ \ddot{\phi} \end{Bmatrix} + 2\omega_o \begin{bmatrix} \zeta & 0 \\ 0 & 4\bar{r}^2\zeta \end{bmatrix} \begin{Bmatrix} \dot{z} \\ \dot{\phi} \end{Bmatrix} + \omega_o^2\{\bar{F}\} = -\begin{Bmatrix} M\omega_o^2 \ddot{u}_g / F_y \\ 0 \end{Bmatrix} \quad (2)$$

where

$z(t) = u_y(t)/U_Y$

$\phi(t) = u_\theta(t)D/(2\, U_Y)$

ω_o = uncoupled lateral frequency = $(K_o/M)^{1/2}$

K_o = initial lateral stiffness of the system

$\bar{r} = r/D$

and $\bar{F}$ is a non-dimensional resisting force vector. In incremental form for numerical integration, it can be written as

$$\{\Delta\bar{F}\} = \begin{bmatrix} \Sigma\, \bar{K}_i(t) & 2\Sigma\, \bar{K}_i(t)\, \bar{x}_i \\ 2\Sigma\, \bar{K}_i(t)\, \bar{x}_i & 4\Sigma\, \bar{K}_i(t)\, \bar{x}_i^{\,2} \end{bmatrix} \begin{Bmatrix} \Delta z \\ \Delta\phi \end{Bmatrix}$$

where

$\bar{K}_i = K_i/K_o$

and

$\bar{x}_i = x_i/D.$

For eq. (2), a viscous damping term is added to account for energy dissipation of the system. 2% critically damped is the damping value taken in this study.

The equations of motion are integrated numerically using step-by-step integration, assuming linear variation of acceleration over a short time interval Δt. To satisfy the stability condition of the numerical method, Δt is taken as 0.02 seconds. In the time increments during which stiffness of any frame changes due to yielding or unloading from yielding, there will be an overshoot of response away from the prescribed load deformation relation, resulting in unbalanced forces during that step. The resulting unbalanced forces are treated using the modified Newton-Raphson iteration method to reduce them to an acceptably small value before the response of the system for the next time step is taken.

PARAMETRIC STUDIES

A parametric study is carried out on the response of the system. The parameters included in the study are:

(i) The eccentricity ratio e/D. The eccentricity ratio takes values of 0, 0.05, 0.15 and 0.25. e/D = 0 denotes the case of a symmetrical structure and forms the basis to evaluate the effect of asymmetry. The other three values represent small, medium and large eccentricity systems respectively. Systems with different values of e/D are obtained by proper choice of the initial stiffness distribution in the resisting elements.

(ii) Uncoupled torsional to lateral frequency ratio Ω. This ratio takes on values equal to 0.8, 1.0 and 1.4. These values of frequency ratio are representative of buildings with resisting elements located close to the core, uniformly distributed in plan, and located at the perimeter of the building respectively. In the model studied, changing of Ω is obtained by varying the distance between the frames $\bar{x}$. The values of $\bar{x}$ used were 0.37, 0.40 and 0.50 respectively for the frequency ratio used.

(iii) Ground Motion Excitation $\ddot{u}_g(t)$. Two ground motion records were used. They were the El Centro 1940 N-S component and the Taft 1952 S69°E component records. For each record, the ground motions is normalized with respect to the 2% damped elastic spectral acceleration value S_a at 1 second period. Writing $\ddot{u}_g(t) = S_a \bar{\ddot{u}}_g(t)$, the excitation term in eq. (2) can be written as $\alpha\, \omega_o^2\, \bar{\ddot{u}}_g(t)$ where $\alpha = M S_a / F_Y$. α is then taken as the excitation level parameter. When α equals unity, it represents the excitation level as such that the system with zero eccentricity just reaches yield. For systems with other values of eccentricity, the amount of yielding at some of the resisting elements is minimal. Hence, responses associated with $\alpha=1$ can be considered as elastic response values. The parameter α is varied from 1 to 8 in this study. High values of α indicate that the system is being excited well into the inelastic range.

DISCUSSION OF RESULTS

There are two response parameters that are of special interests in design. They are the ductility demand in the resisting elements and the edge displacements of the structure. The former parameter is useful in the design and detailing of the resisting elements, and the latter is a good measure of the nonstructural damage potential. In this paper, the ductility demand is presented by the peak ductility ratio μ at the element furthest away from the center of stiffness. It is defined as the ratio of the absolute maximum displacement to the yield displacement of that element. The edge displacement considered is the displacement of the edge of the building furthest away from the center of stiffness. Information on the edge displacement is normalized with respect to the same edge displacement if the system were symmetric.

Shown in Figures 3 and 4 are the results of the ductility ratio μ for element C against the excitation intensity parameter α under El Centro 1940 and Taft 1952 ground motion excitations. The system parameters are the normalized eccentricity e/D and the torsional to lateral frequency ratio Ω. In addition to denote the level of excitation, the parameter $\alpha = MS_a/F_y$ can be considered as the ratio of the elastic force on the system to the yield strength of the system, assuming the system is symmetrical. Therefore, it may be considered as the inverse of the reduction factor R. For a S.D.F. elastoplastic system, the reduction factor R is related to the ductility factor μ by the relation $R = 1/\mu$ if the equal maximum displacement criteria is used; or $R = 1/\sqrt{(2\mu-1)}$ if the equal strain energy criteria is used [6]. Although the ductility ratio μ plotted in Figures 3 and 4 are ductility demand for one resisting element in a two degrees of freedom inelastic system, the curves $\alpha = \mu$ and $\alpha = \sqrt{(2\mu-1)}$ would still be useful guidelines to provide a basis to interpret the results obtained. Therefore, they are plotted on the figures also.

By comparing the results based on the El Centro record versus those based on the Taft record, it is seen that the ductility demand is greatly influenced by the ground motions used. Excitation by the El Centro record leads to a smaller ductility demand than using the Taft record, and the curve $\alpha=\mu$ gives a representative relation of the ductility demand curves for El Centro excitation. Using the Taft record, the ductility demand curves are bounded approximately by the curve $\alpha=\mu$ and $\alpha = \sqrt{(2\mu-1)}$. Obviously, the frequency content in the ground motion used has a significant effect, although such effect is not easy to quantify. Blume [7] has shown that the ductility demands for a number of single degrees of freedom inelastic system under El Centro record excitation are bounded by the same curves $\alpha = \mu$ and $\alpha = \sqrt{(2\mu-1)}$. In this respect, the ductility demand of frame C in our model is similar to many of the single degree of freedom inelastic system studied.

In general, systems with higher initial eccentricity lead to higher ductility demand. In the cases where the Taft record is used, it is not uncommon to have ductility demand of a highly eccentric system being more than twice that of a symmetrical system. In this aspect, our finding differs from the observations made by Irvine and Kountouris [4]. One possible explanation is that a different structural model used in the current study.

The uncoupled torsional to lateral frequency ratio has only minor effect on the ductility demand of frame C. This observation can be model specific rather than generic in nature. In the structural model considered, the increase in torsional stiffness is obtained by locating the frames A and C further away from the center of mass. Therefore, although a torsionally stiffer system ($\Omega = 1.4$) may result in smaller value of rotational motion, the resulting translational motion in frame C is not reduced proportionally since in such a configuration, the lever arm between frame C and the center of stiffness is increased.

Elastic study of torsionally unbalanced system indicates that when the torsional and lateral frequencies are the same ($\Omega = 1.0$), and particularly for systems with small eccentricity, substantial increase in torsional response is expected [1]. However, the case when $\Omega = 1.0$ does not lead to extra large value of ductility demand in the current inelastic response study. This can be due to the fact that due to yielding of the system, the "effective" frequencies of the system are detuned even the initial uncoupled frequencies are equal. At the instant when the peak ductility demand is reached, the system has gone well into the inelastic range and the amplification effect due to coincidence of initial torsional and lateral frequency cannot be felt at such state.

The insensitivity of ductility demand in frame C under El Centro excitation may lead to the wrong conclusion that the overall behaviour of the structure is not much affected by change of eccentricity and torsional to lateral frequency ratio. Shown in Fig. 5 are sketches of the displacements and rotation of the structure at the point the peak ductility demand of frame C is reached, under El Centro ground motion excitation. For a given value of Ω, it is seen that a system with higher eccentricity leads to a larger rotational motion. However, it is compensated with a smaller lateral displacement at the mass center. As a result, the ductility demand on frame C is relatively insensitive to the change of eccentricity, as presented by the current study and also pointed out by Kan and Chopra. The open circle denotes the element is on the yielding branch and the solid circle denotes the element is on the unloading branch of the element force-displacement curve. It can be seen at the moment peak ductility demand is reached, frame C is on the yielding branch while frames A and B are on the unloading branches of their respective curves. This indicates substantial torsional motions is involved at that instant, and does not agree with the observation by Kan and Chopra that when such a system gets into the inelastic range, it behaves more and more like an inelastic single-degree-of-freedom system, responding primarily in translation.

Frame C will be the frame which has the greatest ductility demand if the translational and torsional motion are in phase at the instant of maximum excursion. While this is generally true, there are occasions when the translational and torsional motions become out of phase at the instant of maximum excursion, thus making frame A (the element closer to the stiffness center than frame C) subjected to higher ductility demand. An example of such response is shown in Fig. 5. Currently, there is no clear cut criteria to predict such a condition will occur. Further study on this issue is useful since very little seismic code provisions are directed to the design of elements on the same side as the stiffness center (measured relative to the mass center).

The positive edge displacement response is represented by the edge displacement ratio Δ_r. It is defined as the ratio of the positive edge displacement for systems with eccentricities to the edge displacement of an otherwise similar symmetric system. In other words, it is a measure of the effect of eccentricity on edge displacement response. Shown in

Figures 6 and 7 are the edge displacement ratio as a function of excitation level α for two ground motion records. In general, larger eccentricity tends to lead to larger edge displacement ratio; however, the increase is not systematic and difficult to quantify. Also, Δ_r can be less than unity, indicating the edge displacement of an eccentric system is not necessarily larger than a symmetrical one. The most important variable that affects the edge displacement ratio appears to be the ground motion used. The maximum edge displacement ratio is of the order of two for the El Centro ground record while the maximum value is over three when the Taft record was used. It is of interest to note [1] that such a system under a flat acceleration spectrum input and assuming elastic response, the edge displacement ratio is of the order of two, similar to the inelastic edge displacement ratio produced by El Centro excitation over a range of eccentricity values.

CONCLUSION

Although only the results for the case of structural system with uncoupled lateral period of 1 second are presented, the observations listed below apply to systems with shorter (0.5 second) and longer (2.0 second) lateral periods. Details of the study on these system can be found in ref. [5].

One can make the following observations, based on the results presented: (1) The ductility ratio μ increases with excitation level α. The curve $\alpha = \sqrt{(2\mu-1)}$ appears to be a good upper bound for the ductility demand for any given excitation level α. (2) Eccentricity in the system has a larger effect on the ductility demand than reported earlier [4]. An increase of over 100% in ductility demand is not unusual for systems with large eccentricity as compared to systems with small eccentricity. (3) The coincidence of uncoupled torsional and lateral frequencies does not lead to additional peak inelastic responses, although such condition leads to larger elastic response in general. (4) Sizeable rotational motion is involved at the instant where peak ductility demand is reached. Therefore, an eccentric structure does not respond primarily in translation when it is excited well into the inelastic range as reported by early investigators. (5) Eccentricity has the effect of increasing the edge displacement of the structure by a factor up to three when compared with that of a symmetrical system. Increase in torsional stiffness of the structure tends to reduce this factor as expected. (6) The behaviour, and hence the different peak response parameters are strongly dependent on the ground motions used. Attempts have been made in the current study to normalize the excitation level parameter with the acceleration spectral values at different systems' periods such as the coupled first or second periods instead of the uncoupled lateral periods in order to reduce the dependence of results on different ground motions. The failure of such attempts to reduce the dispersion points to the necessity of additional parameters to quantify the influence of different ground motions on the inelastic response. Until the influence of ground motions on the response parameters is better understood, all conclusions drawn based on studies using a limited number of ground motion records as excitation must be viewed as tentative.

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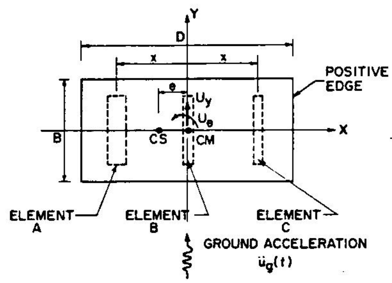
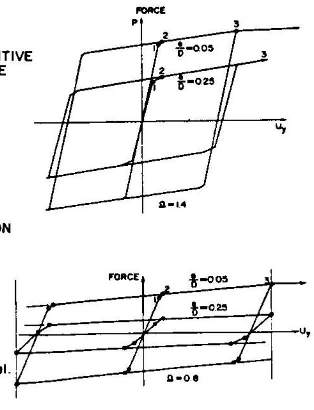




CM — CENTER OF MASS

CS — CENTER OF STIFFNESS

Figure I. Idealized Single Mass Mono Symmetric Model.

Figure 2 Load Displacement Curve System at Center of Mass

○ ECC.=0.0 ▲ ECC.=0.15

□ ECC.=0.05 ◆ ECC.=0.25

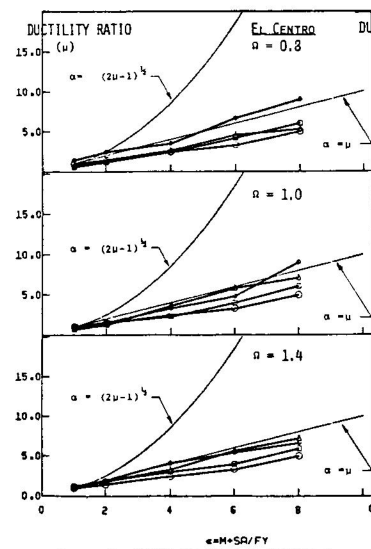


FIGURE 3. DUCTILITY RATIO ON ELEMENT C.

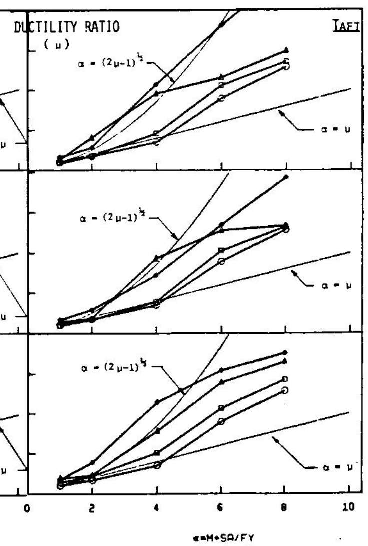


FIGURE 4. DUCTILITY RATIO ON ELEMENT C.

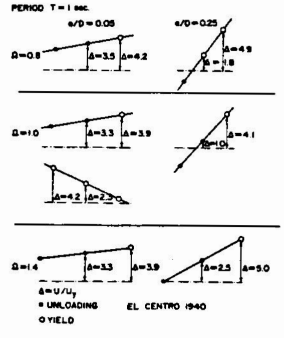
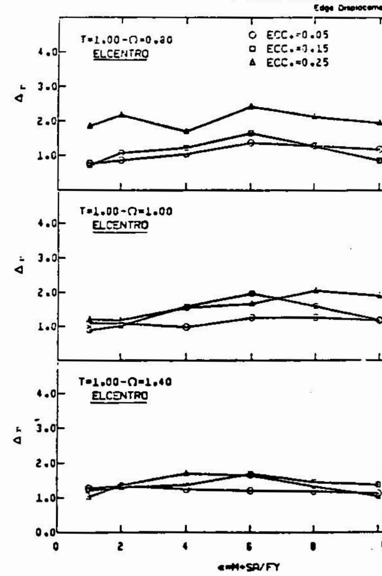


Figure 5. Position of Structure at Instant of Maximum Edge Displacement



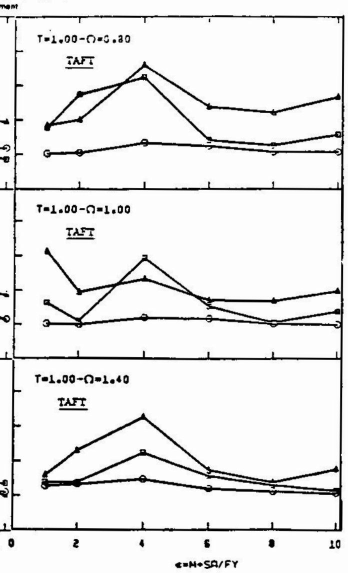


FIGURE 6. VARIATION OF EDGE DISPLACEMENT RATIO FIGURE 7. VARIATION OF EDGE DISPLACEMENT RATIO